Critical Communication Radius for Sink Connectivity in Wireless Networks

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Outlines

- **Introduction**
- **Asymptotic sink connectivity**
- **Critical communication radius for sink connectivity**
- **Effective communication radiuses for different link models**

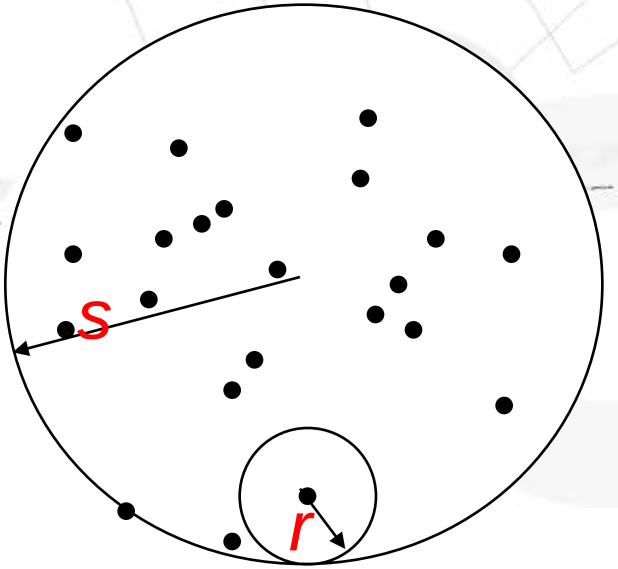
Wireless Sensor Networks

- Small devices with capability of sensing, processing and wireless communication
- Distributed and autonomous wireless networks with self-organization and cooperation for information acquisition
- Wide variety of applications for infrastructure safety, environmental monitoring, manufacturing and production, logistics, health care, security surveillance, target detection/localization/tracking, etc

Challenging problems and issues

- Limited node resources in terms of energy, bandwidth, processing capacity, storage, etc
- Energy consumption $\propto$ {processing speed$^{2\text{-}4}$, sensing radius$^{q=2\text{-}4}$, communication radius$^{q=2\text{-}4}$}
- Energy constrained communication protocol
- Special issues on connectivity, time synchronization, localization, sensing coverage, task allocation, data management, etc.

Connectivity problem



$G(n, s, r)$: the network in consideration

s : disc radius

A: disc area, $A = \pi s^2$

r : communication radius, if $\|x_i - x_j\| < r$, i→j and j→i

n : the number of nodes

d : $d = n\pi r^2 / A$, average number of neighbor nodes

Determine the minimal *r* to guarantee the connectivity of the network

Existing result
(P. Gupta and P. R. Kumar,1998)

- **Critical radius for fully connected graph (no isolated node)**

The network is asymptotically ($n \rightarrow \infty$) fully connected with probability one if and only if

$$r = \sqrt{A(\log n + \gamma) / \pi n}$$

with variable $\gamma \rightarrow \infty$

Issue

- **Full connection may not be necessary for some applications**
- **To save energy and prolong lifetime, a very small fraction isolated nodes of in a wireless sensor network with thousands of nodes could be tolerated**

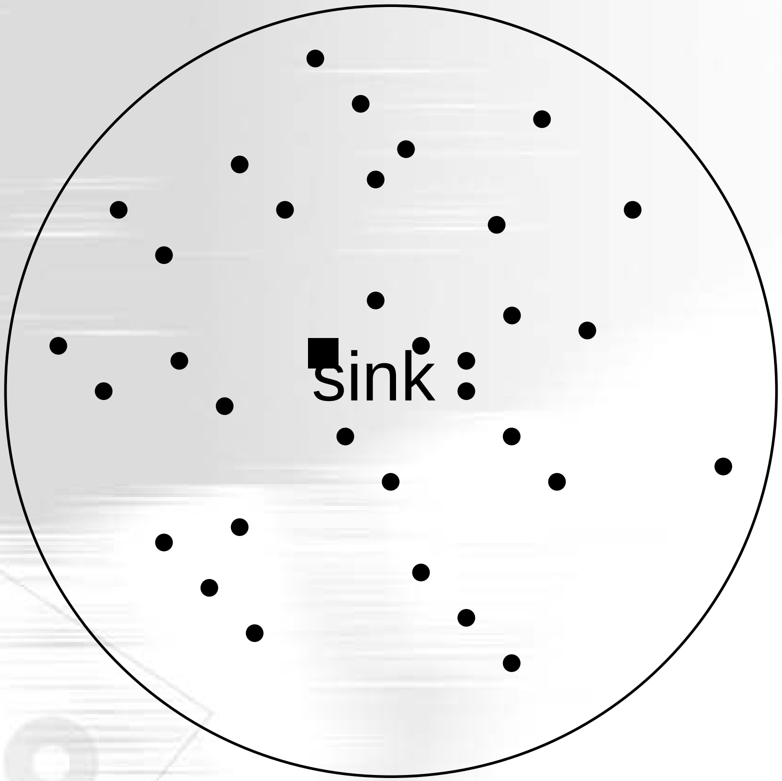
Introducing sink connectivity

- **Assume the sink is a randomly selected node in the network**
- **Sink connectivity C_n is defined as the fraction of nodes in the network that are connected to the sink**

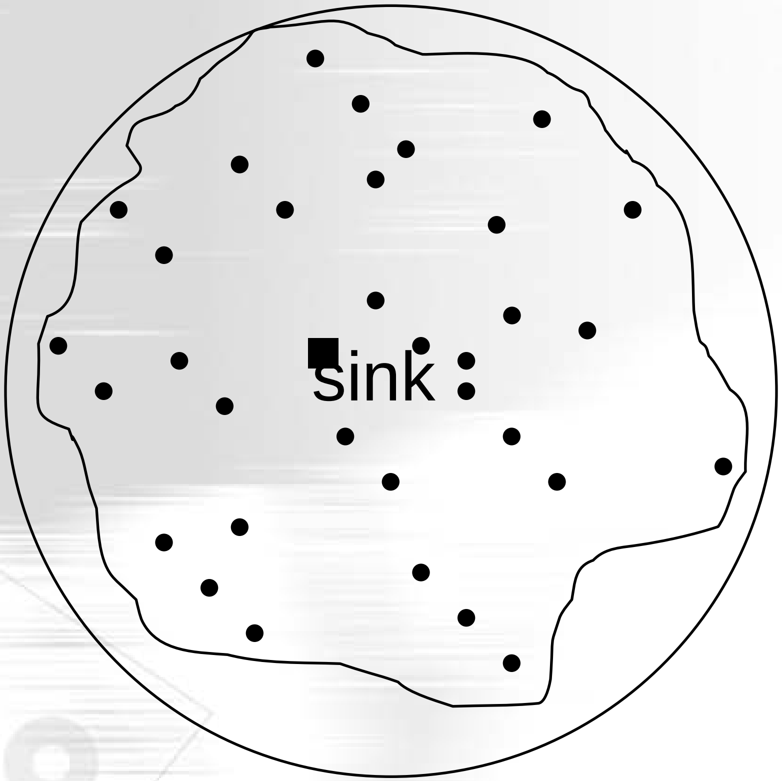
Goal

- **Find the critical communication radius to guarantee $Cn > \alpha$, where α is a constant close to 1**

Connected subnet



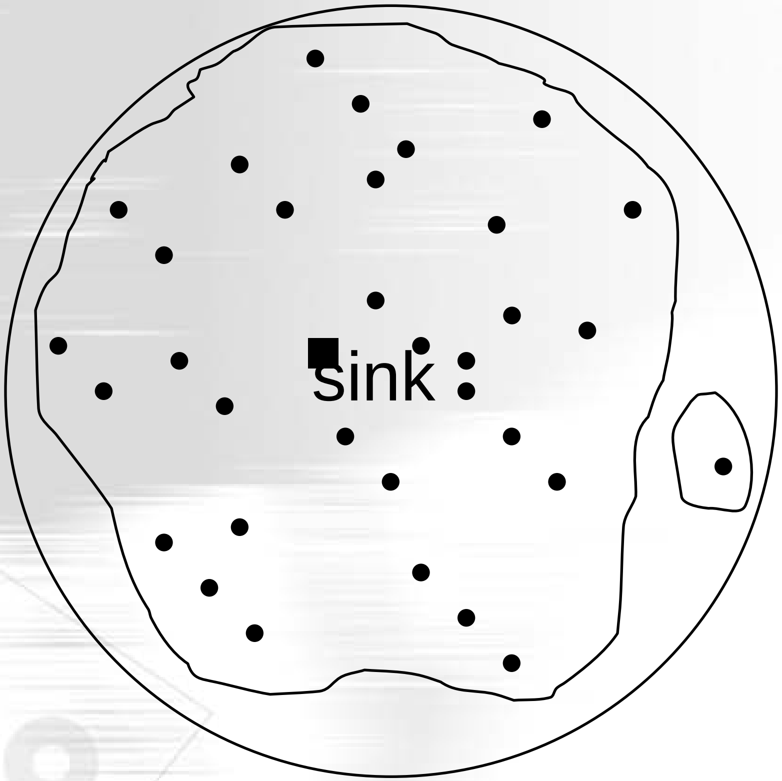
Let $L_j(n,s,r)$ be the number of nodes in the *j*th-largest connected subnet in $G(n,s,r)$



Fully connected

$\Leftrightarrow$ $C_n=1$

$\Leftrightarrow$ $L_1(n,s,r)=n$



Partial connected

$\Leftrightarrow C_n < 1$

$\Leftrightarrow L_1(n,s,r) < n$

The expectation of C_n

$$E(C_n) = \sum_i \left(\frac{L_i(n,s,r)}{n}\right)^2$$

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Asymptotic sink connectivity

Based on the continuum percolation theory*, we can get the following two theorems

Theorem 1: If $r_n = \sqrt{Ad/(\pi n)}$, let $d_c = \pi\lambda_c$, then $\forall d < d_c$, $C_n \xrightarrow{P} 0$ as $n \to \infty$.

Theorem 3: If $r_n = \sqrt{Ad/(\pi n)}$, then, $C_n \xrightarrow{P} 1$ as $n \to \infty$ if and only if $d \to \infty$.

* M. Penrose, *Random Geometric Graphs*. New York: Oxford University Press, 2003.

Comparison with the existing result

Gupta's conclusion

Goal

$P_c = \mathbb{P}[C_n = 1] \to 1$ **as** $n \to \infty$

Critical radius:

$r^* = \sqrt{A(\log n + \gamma)/(\pi n)}$

where $\gamma \to \infty$

Example:

$$r^* = \sqrt{A(\log n + \log\log n)/(\pi n)}$$

Current result

Goal

$C_n \xrightarrow{P} 1$ **as** $n \to \infty$

Requirement:

$r = \sqrt{Ad/(\pi n)}$

where $d \to \infty$

Example:

$$r = \sqrt{A(\log\log n)/(\pi n)}$$

Average neighbor number *d*

- **d:** $d = n\pi r^2 / A$
- **Mapping:** $G(n, s, r) \xrightarrow{\times\lambda} G(n, \lambda s, \lambda r)$
  The connectivity is unchanged;
  $d = n\pi r^2 / A$ **is unchanged.**
- **Instead of r, we discuss the relationship between the connectivity and d for simplify.**
- **Using** $r = \sqrt{Ad/(\pi n)}$**, we can get the corresponding communication radius.**

Connectivity versus average number of neighbors

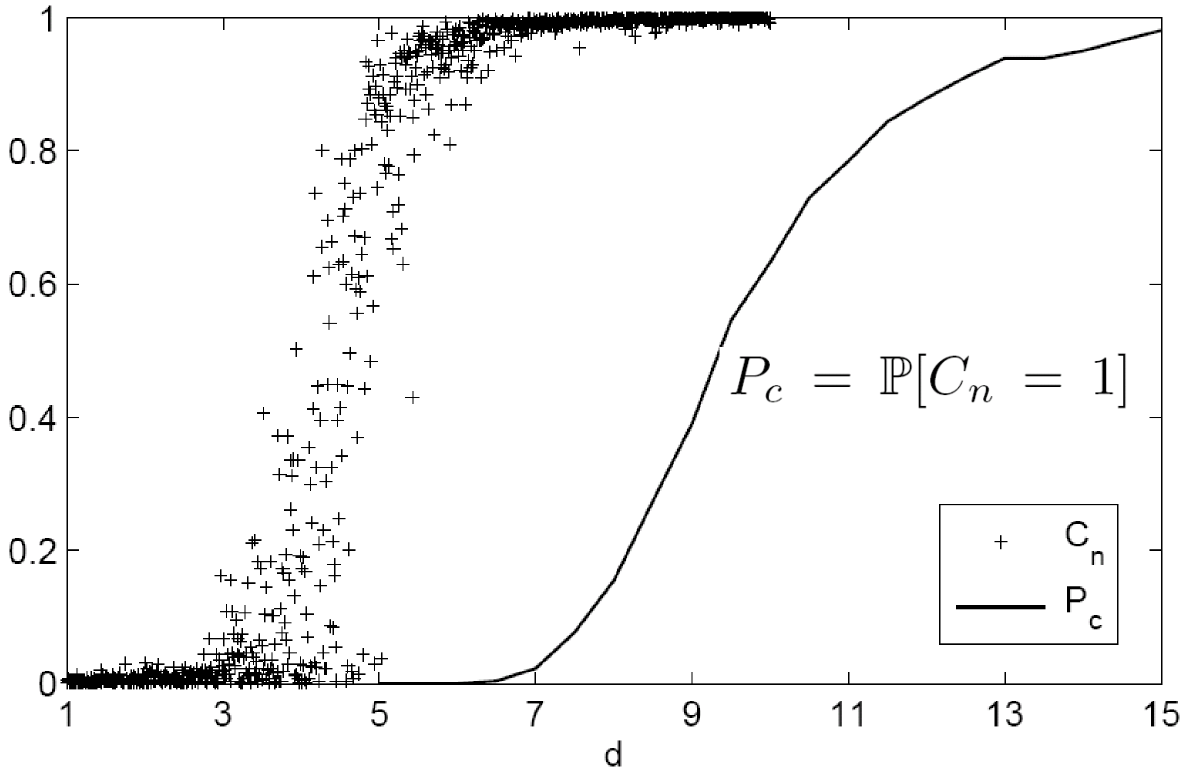

Fig. 2. relations between C_n, P_c and d.

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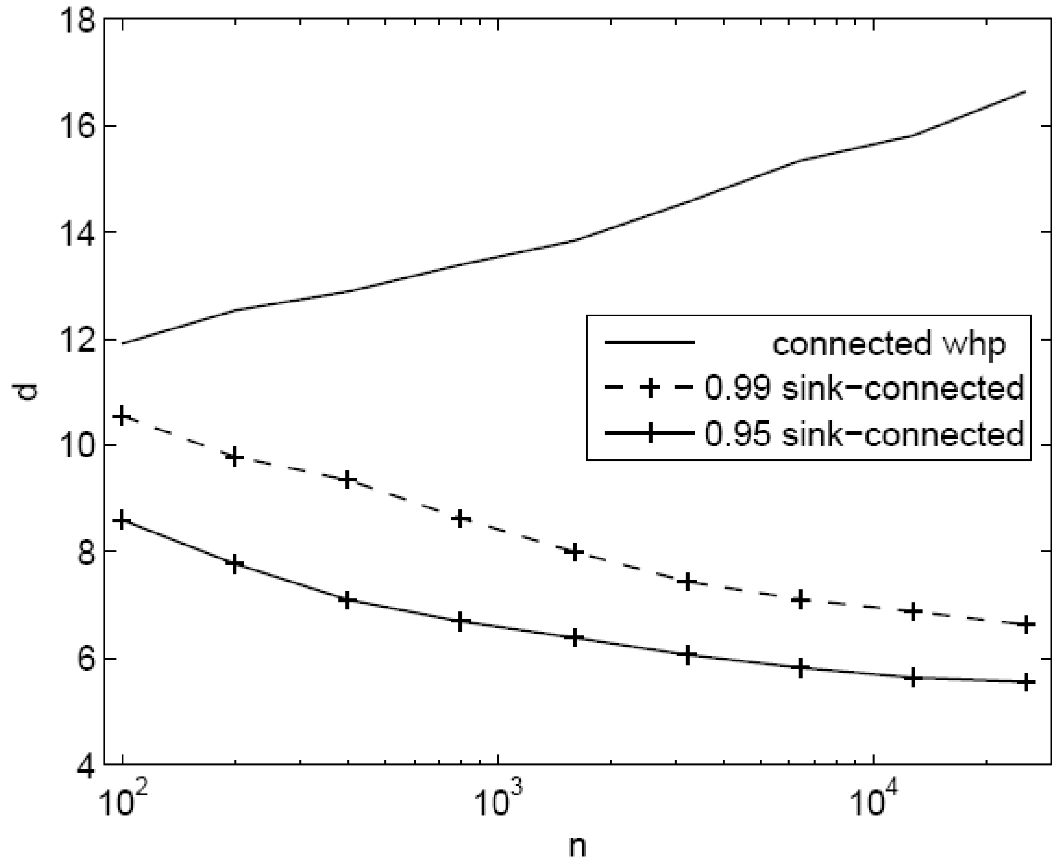
α sink connected

- **A network is α sink connected if $C_n \geq \alpha$ with high probability.**
- **The minimal radius that makes the network α sink connected is the critical communication radius for α sink connected**

Required average neighbor number versus *n*



Critical radius

$$r = \sqrt{Ad/(\pi n)}$$

Fig. 3. Relations between d and n for different levels of connectivity.

Observations

- If we tolerate a small percent of nodes being isolated, the critical communication radius will be considerable reduced.
- This could resulting in reducing communication energy consumption significantly since energy $\propto$ {communication radius$^{q=2-4}$}

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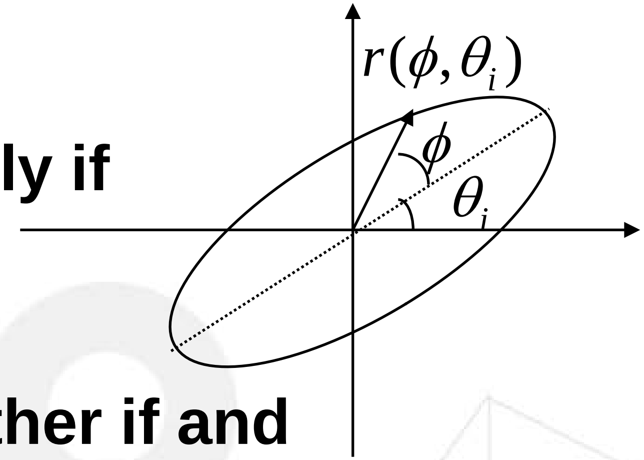
Link models

- **Simple Boolean**

  (x_i, x_j) **can communication with each other if and only if** $\|x_i - x_j\| < r$ **, where r is a constant.**
- **Random connection**

  x_i **can send a message to** x_j **with the probability** $g(\|x_i - x_j\|)$
- **Anisotropic**

  x_i **can send a message to** x_j **if and only if** $\|x_i - x_j\| < r(\phi, \theta_i)$ **, see the figure.**



- **Random radius**

  (x_i, x_j) **can communication with each other if and only if** $\|x_i - x_j\| < r_i$**, where** r_i **is a random variable.**

Effective Communication Radius

- $$r_e = E(\sqrt{e(g)/\pi}) = E(\sqrt{\int_{x \in \mathbb{R}^2} g(x)dx/\pi}).$$

  as the effective communication radius where (x_i, x_j) is connected with probability $g(x_i - x_j)$, $e(g)$ is the effective communication area Numerous of simulation results show that
- **If the effective communication radius > R, the sink connectivity of three other link models (or the combination of three other link models) is better than that of the simple Boolean model**
- **Note: Here the sink connectivity is the fraction of nodes that can receive the broadcasting messages from the sink.**

Average connectivity for different link models

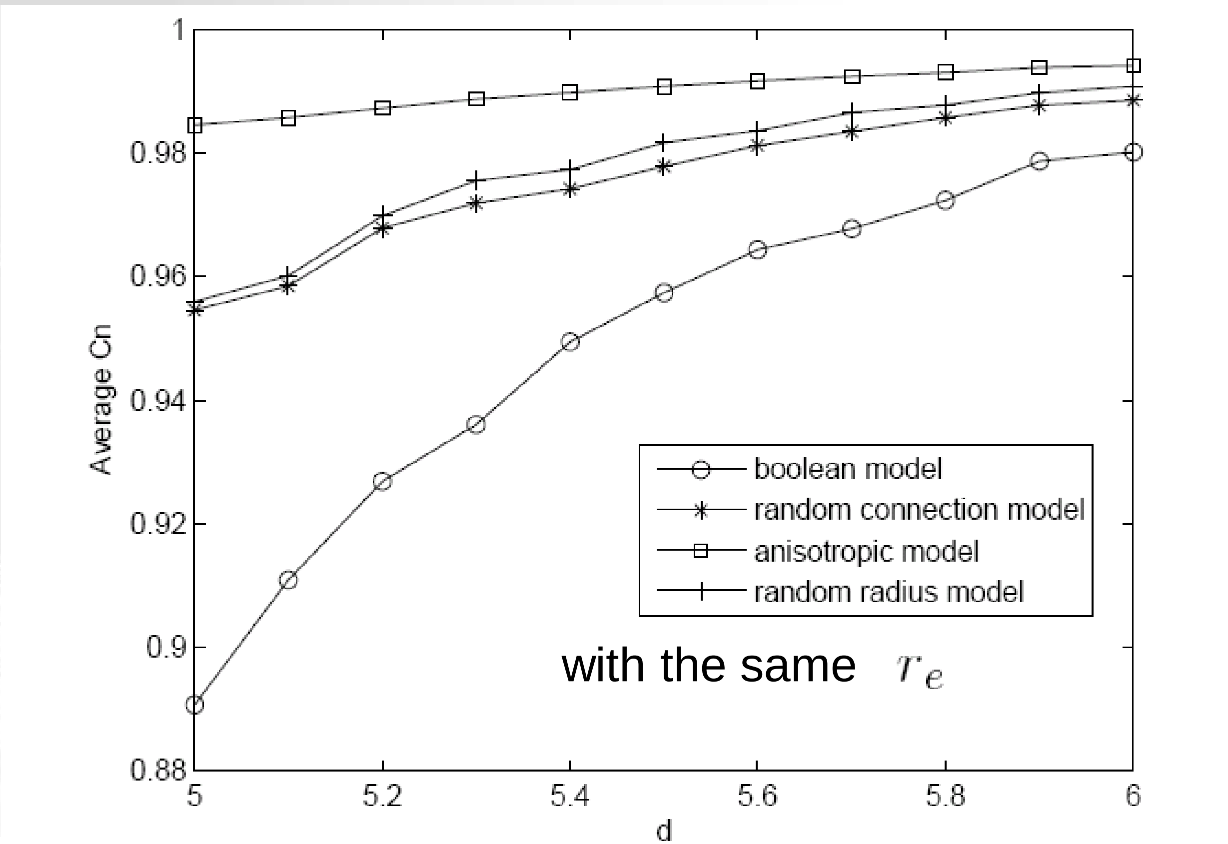


Fig. 4. relations between C_n and d in four different models.

Summary and conclusions

- **Sink connectivity is proposed for wireless sensor networks**
- **If we tolerate a small fraction of nodes being isolated, we can reduce the communication radius, and thus the communication power consumption significantly.**
- **If the density of the nodes remain unchanged, the critical communication radius for sink connectivity would decrease opposite to the critical communication radius for full connectivity.**
- **Effective communication radius is introduced to describe the sink connectivity in more complicated link models.**

- Thank you